Note: In this problem set, expressions in green cells match corresponding expressions in the text answers.

```
Clear["Global`*"]
```

1. In rolling 3 fair dice, what is the probability of obtaining a sum not greater than 16?

I got help from the post of Zihu Guo on *https://mathematica.stackexchange.com/questions/86026/rolling-a-sum-of-6-or-less-on-3-dice*

```
allpos = Tuples[{Range[6], Range[6], Range[6]}];
select = Cases[allpos, a_ /; Total[a] ≤ 16];

prob = Length[select] / Length[allpos]
```

$$\frac{53}{54}$$

```
N[%]
```

```
0.981481
```

3. Three screws are drawn at random from a lot of 100 screws, 10 of which are defective. Find the probability of the event that all 3 screws drawn are nondefective, assuming that we draw (a) with replacement, (b) without replacement.

(a) In the first part I replace after drawing, then draw again. So the probability will be as follows:

```
d = MultivariateHypergeometricDistribution[1, {90, 10}]
MultivariateHypergeometricDistribution[1, {90, 10}]

Probability[x == 1 && y == 0, {x, y} ≈ d]
```

$$\frac{9}{10}$$

```
%^3
```

$$\frac{729}{1000}$$

(b) In the second part I draw without replacement.

```
d = MultivariateHypergeometricDistribution[3, {90, 10}]
MultivariateHypergeometricDistribution[3, {90, 10}]

Probability[x == 3 && y == 0, {x, y} ≈ d]
```

$$\frac{178}{245}$$

```
N[%]
```

```
0.726531
```

5. If a box contains 10 left-handed and 20 right-handed screws, what is the probability of obtaining at least one right-handed screw in drawing 2 screws with replacement?

```
Clear["Global`*"]
```

```
d = MultivariateHypergeometricDistribution[1, {10, 20}]
MultivariateHypergeometricDistribution[1, {10, 20}]
```

I check the probability of getting a rh screw the first time.

```
Probability[x == 0 && y == 1, {x, y} ≈ d]
```

$$\frac{2}{3}$$

And the probability of getting rh screws both times.

```
%^2
```

$$\frac{4}{9}$$

Since I only need to get one rh out of two tries, my chances of getting at least one are double that.

```
% 2
```

$$\frac{8}{9}$$

7. Under what conditions will it make practically no difference whether we sample with or without replacement?

If the sample draw is small compared with the sample from which it is drawn.

9. If we inspect photocopy paper by randomly drawing 5 sheets without replacement from every pack of 500, what is the probability of getting 5 clean sheets although 0.4% of the sheets contain spots?

```
Clear["Global`*"]
```

```
d = MultivariateHypergeometricDistribution[5, {498, 2}]
MultivariateHypergeometricDistribution[5, {498, 2}]
```

```
Probability[x == 5 && y == 0, {x, y} ≈ d]
```

$$\frac{24\,453}{24\,950}$$

```
N[%]
```

```
0.98008
```

11. A batch of 200 iron rods consists of 50 oversized rods, 50 undersized rods, and 100 rods of the desired length. If two rods are drawn at random without replacement, what is the probability of obtaining (a) two rods of the desired length, (b) exactly one of the desired length, (c) none of the desired length?

```
Clear["Global`*"]
```

```
d = MultivariateHypergeometricDistribution[2, {100, 50, 50}]
```

```
MultivariateHypergeometricDistribution[2, {100, 50, 50}]
```

(a) The probability of getting both rods the correct length.

```
Probability[x == 2 && y == 0 && z == 0, {x, y, z} ≈ d]
```

$$\frac{99}{398}$$

```
N[%]
```

```
0.248744
```

(b) The probability of getting exactly one right and one wrong length. The y cache and z cache are the same number, so I can just double the likelihood of the trial.

```
Probability[x == 1 && y == 1 && z == 0, {x, y, z} ≈ d]
```

$$\frac{50}{199}$$

```
N[%]
```

```
0.251256
```

```
2 %
```

```
0.502513
```

(c) The unfruitful draw has three possible aspects, all of which need to be checked.

```
pa = Probability[x == 0 && y == 1 && z == 1, {x, y, z} ≈ d]
```

$$\frac{25}{199}$$

```
pb = Probability[x == 0 && y == 2 && z == 0, {x, y, z} ≃ d]
```

$$\frac{49}{796}$$

```
pc = Probability[x == 0 && y == 0 && z == 2, {x, y, z} ≃ d]
```

$$\frac{49}{796}$$

```
N[pa + pb + pc]
```

```
0.248744
```

13. A pressure control apparatus contains 3 electronic tubes. The apparatus will not work unless all tubes are operative. If the probability of failure of each tube during some interval of time is 0.04, what is the corresponding probability of failure of the apparatus?

```
Clear["Global`*"]
```

Using the help of mathman in post at *https://www.physicsforums.com/threads/probability-over-a-period-of-time.215972/*.

If the probability of one burned tube over the time period of interest is 0.04 then the probability of that tube ***not*** failing is

```
1 - 0.04
```

```
0.96
```

and since all three tubes have the same chance of failing, the chance of the three of them ***not*** failing over the time period of interest is

```
%^3
```

```
0.884736
```

which means that the chance for at least one of them ***to*** fail over that time period is

```
1 - %
```

```
0.115264
```

I also see that Mathematica can take care of this neatly with a binomial probability calculation. The time period isn't considered, but whatever the time period is, over this time period there are considered to be three chances to fail, one for each tube, and because they each have the same chance of failing, I can put into one expression. (The ≥ sign on x is important.)

```
Probability[x ≥ 1, x ≃ BinomialDistribution[3, 0.04]]
```

```
0.115264
```

15. What gives the greater probability of hitting at least once: (a) hitting with probability 1/2 and firing 1 shot, (b) hitting with probability 1/4 and firing two shots, (c) hitting with probability 1/8 and firing 4 shots? First guess.

This one was hard for me, I didn't see the significance of the ≥ sign for a long time. It makes sense that hitting "at least once" is more likely than hitting "exactly once".

```
Probability[x == 1, x ≈ BinomialDistribution[1, 0.5]]
```

```
0.5
```

```
Probability[x ≥ 1, x ≈ BinomialDistribution[2, 0.25]]
```

```
0.4375
```

```
Probability[x ≥ 1, x ≈ BinomialDistribution[4, 0.125]]
```

```
0.413818
```

17. Show that if B is a subset of A, then P(B) ≤ P(A).